GRE MATH Practice Paper 1

1. The average (arithmetic mean) high temperature for *x* days is 70 degrees. The addition of one day with a high temperature of 75 degrees increases the average to 71 degrees.

Quantity A	**Quantity B**
x	5

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

2.

a and *b* are integers.

$a^2 = b^3$

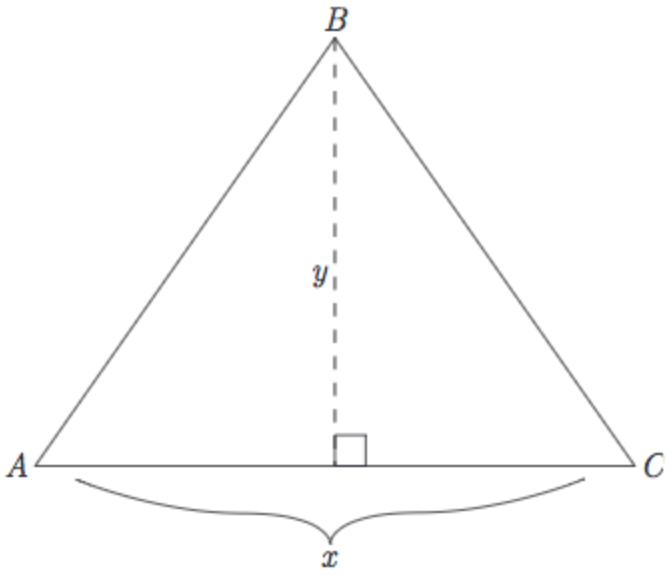
Quantity A	**Quantity B**
a	*b*

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

3. A certain pet store sells only dogs and cats. In March, the store sold twice as many dogs as cats. In April, the store sold twice the number of dogs that it sold in March, and three times the number of cats that it sold in March. If the total number of pets the store sold in March and April combined was 500, how many dogs did the store sell in March?

(A) 80
(B) 100
(C) 120
(D) 160
(E) 180

4.



△ *ABC* has an area of 108 cm 2 . If both *x* and *y* are integers, which of the following could be the value of *x* ?

Indicate all such values.

(A) 4
(B) 5
(C) 6
(D) 8
(E) 9

5. Each month, Renaldo earns a commission of 10.5% of his total sales for the month, plus a salary of $2,500. If Renaldo earns $3,025 in a certain month, what were his total sales?

6. At a recent dog show, there were 5 finalists. One of the finalists was awarded "Best in Show" and another finalist was awarded "Honorable Mention." In how many different ways could the two awards be given out?

7.

$$\lim_{x\to 0}\frac{\cos(3x)-1}{x^2}=$$

(A) $\frac{9}{2}$ (B) $\frac{3}{2}$ (C) $-\frac{2}{3}$ (D) $-\frac{3}{2}$ (E) $-\frac{9}{2}$

8.

What is the area of an equilateral triangle whose inscribed circle has radius 2 ?

(A) 12 (B) 16 (C) $12\sqrt{3}$ (D) $16\sqrt{3}$ (E) $4\left(3+2\sqrt{2}\right)$

9.

$$\int_{e^{-3}}^{e^{-2}} \frac{1}{x \log x}\, dx =$$

(A) 1 (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\log\left(\frac{2}{3}\right)$ (E) $\log\left(\frac{3}{2}\right)$

10-12

. Let V and W be 4-dimensional subspaces of a 7-dimensional vector space X. Which of the following CANNOT be the dimension of the subspace $V \cap W$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

. Sofia and Tess will each randomly choose one of the 10 integers from 1 to 10. What is the probability that neither integer chosen will be the square of the other?

(A) 0.64 (B) 0.72 (C) 0.81 (D) 0.90 (E) 0.95

. Which of the following shows the numbers $2^{1/2}$, $3^{1/3}$, and $6^{1/6}$ in increasing order?

(A) $2^{1/2} < 3^{1/3} < 6^{1/6}$

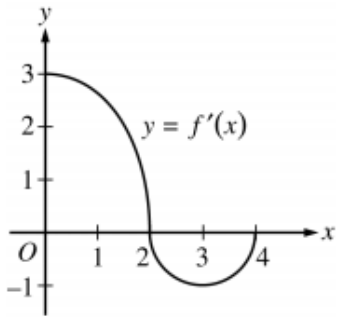
(B) $6^{1/6} < 3^{1/3} < 2^{1/2}$

(C) $6^{1/6} < 2^{1/2} < 3^{1/3}$

(D) $3^{1/3} < 2^{1/2} < 6^{1/6}$

(E) $3^{1/3} < 6^{1/6} < 2^{1/2}$

13.



'. The figure above shows the graph of the derivative f' of a function f, where f is continuous on the interval $[0, 4]$ and differentiable on the interval $(0, 4)$. Which of the following gives the correct ordering of the values $f(0)$, $f(2)$, and $f(4)$?

(A) $f(0) < f(2) < f(4)$

(B) $f(0) < f(4) = f(2)$

(C) $f(0) < f(4) < f(2)$

(D) $f(4) = f(2) < f(0)$

(E) $f(4) < f(0) < f(2)$

14.

Which of the following is NOT a group?

(A) The integers under addition

(B) The nonzero integers under multiplication

(C) The nonzero real numbers under multiplication

(D) The complex numbers under addition

(E) The nonzero complex numbers under multiplication

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Let g be a continuous real-valued function defined on $\mathbb{R}$ with the following properties.

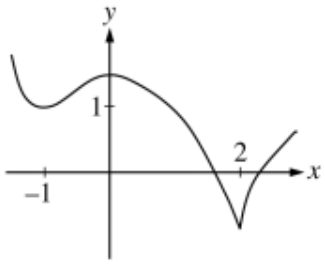
$$g'(0) = 0$$

$$g''(-1) > 0$$

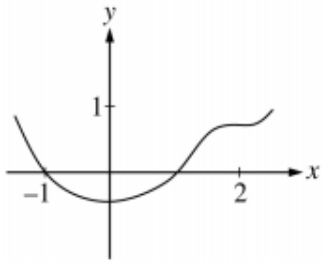
$$g''(x) < 0 \text{ if } 0 < x < 2.$$

Which of the following could be part of the graph of g ?

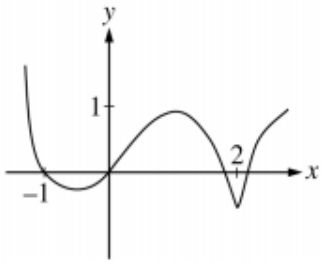
(A)



(B)



(C)



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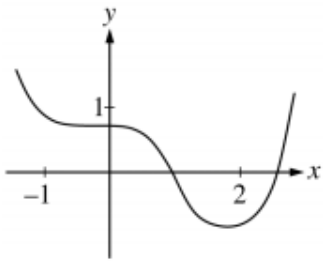
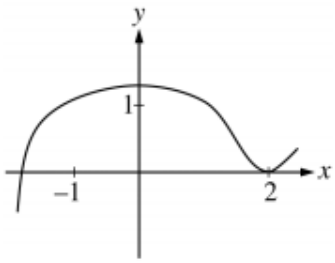


(E)



16.

$$\sqrt{(x+3)^2 + (y-2)^2} = \sqrt{(x-3)^2 + y^2}$$

In the xy-plane, the set of points whose coordinates satisfy the equation above is

(A) a line (B) a circle (C) an ellipse (D) a parabola (E) one branch of a hyperbola

17-19

The region bounded by the curves $y = x$ and $y = x^2$ in the first quadrant of the xy-plane is rotated about the y-axis. The volume of the resulting solid of revolution is

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$ (E) $\frac{3\pi}{2}$

For which integers n such that $3 \leq n \leq 11$ is there only one group of order n (up to isomorphism) ?

(A) For no such integer n

(B) For 3, 5, 7, and 11 only

(C) For 3, 5, 7, 9, and 11 only

(D) For 4, 6, 8, and 10 only

(E) For all such integers n

If f is a continuously differentiable real-valued function defined on the open interval $(-1, 4)$ such that $f(3) = 5$ and $f'(x) \geq -1$ for all x, what is the greatest possible value of $f(0)$?

(A) 3 (B) 4 (C) 5 (D) 8 (E) 11

Suppose g is a continuous real-valued function such that $3x^5 + 96 = \int_c^x g(t)\,dt$ for each $x \in \mathbb{R}$, where c is a constant. What is the value of c ?

(A) –96 (B) –2 (C) 4 (D) 15 (E) 32

Let S, T, and U be nonempty sets, and let $f : S \to T$ and $g : T \to U$ be functions such that the function $g \circ f : S \to U$ is one-to-one (injective). Which of the following must be true?

(A) f is one-to-one.

(B) f is onto.

(C) g is one-to-one.

(D) g is onto.

(E) $g \circ f$ is onto.

Suppose A, B, and C are statements such that C is true if exactly one of A and B is true. If C is false, which of the following statements must be true?

(A) If A is true, then B is false.

(B) If A is false, then B is false.

(C) If A is false, then B is true.

(D) Both A and B are true.

(E) Both A and B are false.

Which of the following equations has the greatest number of real solutions?

(A) $x^3 = 10 - x$

(B) $x^2 + 5x - 7 = x + 8$

(C) $7x + 5 = 1 - 3x$

(D) $e^x = x$

(E) $\sec x = e^{-x^2}$

Let f be the function defined by $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n}$ for all x such that $-1 < x < 1$. Then $f'(x) =$

(A) $\frac{1}{1-x}$ (B) $\frac{x}{1-x}$ (C) $\frac{1}{1+x}$ (D) $\frac{x}{1+x}$ (E) 0

If z is a complex variable and $\overline{z}$ denotes the complex conjugate of z, what is $\lim_{z \to 0} \frac{(\overline{z})^2}{z^2}$?

(A) 0 (B) 1 (C) i (D) ∞ (E) The limit does not exist.

. Let g be the function defined by $g(x) = e^{2x+1}$ for all real x. Then $\lim_{x \to 0} \dfrac{g(g(x)) - g(e)}{x} =$

(A) $2e$ (B) $4e^2$ (C) e^{2e+1} (D) $2e^{2e+1}$ (E) $4e^{2e+2}$

. What is the value of $\int_{-\pi/4}^{\pi/4} \left(\cos t + \sqrt{1+t^2}\,\sin^3 t \cos^3 t\right) dt$?

(A) 0 (B) $\sqrt{2}$ (C) $\sqrt{2} - 1$ (D) $\dfrac{\sqrt{2}}{2}$ (E) $\dfrac{\sqrt{2} - 1}{2}$

. What is the volume of the solid in xyz-space bounded by the surfaces $y = x^2$, $y = 2 - x^2$, $z = 0$, and $z = y + 3$?

(A) $\dfrac{8}{3}$ (B) $\dfrac{16}{3}$ (C) $\dfrac{32}{3}$ (D) $\dfrac{104}{105}$ (E) $\dfrac{208}{105}$

. Let $(\mathbb{Z}_{10}, +, \cdot)$ be the ring of integers modulo 10, and let S be the subset of $\mathbb{Z}_{10}$ represented by $\{0, 2, 4, 6, 8\}$. Which of the following statements is FALSE?

(A) $(S, +, \cdot)$ is closed under addition modulo 10.

(B) $(S, +, \cdot)$ is closed under multiplication modulo 10.

(C) $(S, +, \cdot)$ has an identity under addition modulo 10.

(D) $(S, +, \cdot)$ has no identity under multiplication modulo 10.

(E) $(S, +, \cdot)$ is commutative under addition modulo 10.

. Consider the system of linear equations

$$\begin{aligned} w + 3x + 2y + 2z &= 0 \\ w + 4x + y &= 0 \\ 3w + 5x + 10y + 14z &= 0 \\ 2w + 5x + 5y + 6z &= 0 \end{aligned}$$

with solutions of the form (w, x, y, z), where w, x, y, and z are real. Which of the following statements is FALSE?

(A) The system is consistent.

(B) The system has infinitely many solutions.

(C) The sum of any two solutions is a solution.

(D) $(-5, 1, 1, 0)$ is a solution.

(E) Every solution is a scalar multiple of $(-5, 1, 1, 0)$.